Monocular Volumetric Reconstruction of a Fluid Flow Based on Image Registration and Molecular Tagging

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1 Overview

We propose to combine image registration and volumetric reconstruction from a monocular video of a Hele-Shaw (HS) cell being drained. An HS cell is a tank whose thickness is small (*e.g.* 1 mm) compared to the other dimensions (*e.g.* 400×800 mm^2). A pattern is tagged in the liquid at a molecular level by *photobleaching* [1]. The evolution of the pattern is filmed with a camera whose principal axis is orthogonal to the wall of the cell. Classical methods cannot be used directly to track the pattern because what is observed is the integration of the marked molecules over the entire thickness of the cell and because the velocity of the fluid is not uniform across the thickness of the cell. The proposed approach extends classical direct image registration (DIR) [2] by incorporating a volumetric image formation model and simple constraints based on the geometry of the experimental setup. It allows us to accurately measure the motion and the velocity profiles for the entire volume which is something usually hard to achieve. The results we obtain are consistent with the theoretical hydrodynamic behaviour for this flow which is known as the *planar Poiseuille flow* (PPF).

The interest of this work is twofold. In computer vision, it is a new algorithm for monocular volumetric reconstruction. In fluid mechanics, it allows us to experimentally measure the PPF in a narrow confined environment and it is a method based on two new elements: photobleaching and DIR.

Computer vision algorithms are mainly dedicated to opaque objects. A review of algorithms for transparent objects is given in [3]. A work sharing similarities with our is [4] where the authors proposes a variant of optical flow estimation from images that uses a volumetric model of a liquid in a microchannel. However, contrarily to our approach, [4] uses the PPF as a strong assumption.

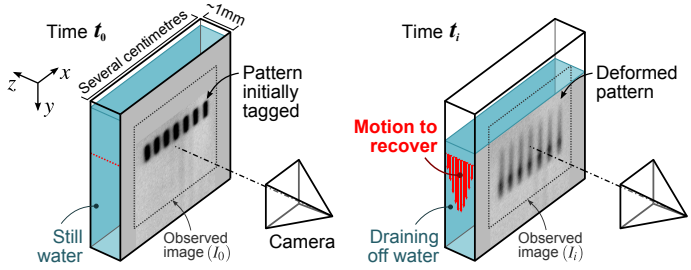

Fig. 1. We propose a new approach to register 2D images of a HS cell being drained while simultaneously recovering the 3D motion of the fluid from a single video. Our approach extends DIR [2]. It uses as few physical assumptions as possible so as to experimentally validate the theoretical flow model.

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2 Experimental Setup

The HS Cell. The HS cell we used for this paper is an almost bi-dimensional cell made of two parallel plates of glass (400×800 mm^2) separated by a 1 mm wide gap (figure 2). The cell we used is the same as the one used in [5]. The cell is filled with water and a downward translation is generated by draining off the cell with a valve located at the bottom. The liquid's motion is filmed at 250 Hz from a fronto-parallel point of view with a fast camera (Photron RS3000) at a resolution of 1024 pixels^2.

Molecular Tagging Based on Photobleaching. In order to observe the fluid motion, we tagged the liquid at the molecular level with a technique known as *photobleaching* [1]. It has the advantage of being truly non-invasive contrarily to other techniques such as PIV where particles are added (which can have their proper motion and may adhere to the cell's walls). The photobleaching method is an irreversible photochemical reaction suitable for the long time scale of our experiment. Its general principle is as follows. A tracer, the *fluorescein*, is uniformly mixed to the water (at a concentration of 10^{-4} g.L^{-1}). First, the fluorescein is marked with a 1 W laser of wavelength $\lambda_1 = 488$ nm. When this lighting is maintained long enough, the properties of the fluorescein molecules are irreversibly inhibited by photobleaching [1]. Then, a second laser beam of wavelength $\lambda_2 = 527$ nm excites the unmarked fluorescein with a power of 15 W. When excited with such a light source the tracer emits light by fluorescence (note that the whole volume emits light). A set of lens and mirrors are used to mark a pattern in the liquid (see figure 2). Two examples of acquired images are shown in figure 1.

Image Formation. The intensity of a pixel is linked to the concentration of non-bleached fluorescein. The important thing here is that the intensity of a pixel is the concentration integrated on the small volume of the cell that corresponds to this pixel. This is illustrated in figure 2c.

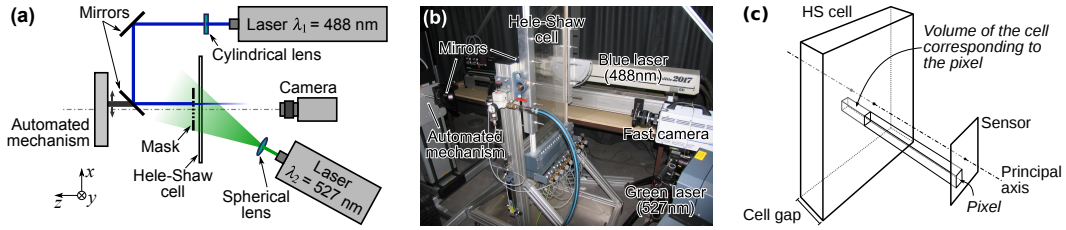

Fig. 2. (a) Schematic of our setup (top view). (b) Front view of our setup. (c) The intensity value of a pixel is proportional to the quantity of non-inhibited fluorescein in the volume of the cell corresponding to that pixel.

3 Simultaneous Registration and Volumetric Reconstruction

Direct Image Registration. Image registration is the problem of finding the geometric transformation between two images. In DIR [2], this transformation is determined directly from the intensities of the pixels. Let R be a reference image (in our case, an image of the bleached pattern before any motion) and let I_i be the ith image of the input video ($i = 1, \ldots, m$). If we naively consider that the motion is a global downward translation of magnitude t, then aligning I_i to R with DIR is formulated as:

$$\min_t \sum_{(x,y)\in\Omega} \left(I_i(x,y) - R\left(x, y-t\right)\right)^2, \tag{1}$$

where Ω is the *region of interest i.e.* a part of I_i containing the bleached pattern. However, formulation (1) is valid only if the colour of corresponding pixels are identical. In our case, the intensity

of a pixel is the integral of the corresponding cell's volume. Since the motion of the fluid along the z-direction is not uniform, there is no direct colour correspondence between a pixel of the reference image and the corresponding pixels in the input video. Consequently, the motion of the marked pattern cannot be explained with a simple translation.

The Proposed Approach. The key idea of our approach is to combine DIR [2] with a deformation model accounting for the underlying physical model of the flow. To do so, we consider that the volume is divided into n layers L_j parallel to the cell walls as illustrated in figure 3 ($L_j(x,y) = R(x,y)/n$ and $R(x,y) = \sum_{j=1}^{n} L_j(x,y)$). A translation parameter $t_{i,j}$ is associated to the jth layer ($j = 1,\ldots,n$) for the ith image ($i = 1,\ldots,m$) of the input video. These translations are determined by minimizing a cost function inspired by the one classically used in DIR. In addition to this layered model, we include some supplementary constraints on the translations:

- **Symmetry.** The flow is symmetric with respect to the centre of the cell in the z-direction. This is naturally enforced by modelling only half the cell.
- **Positivity.** All the translations are downward translations ($t_{i,j} \geq 0$).
- **Temporal consistency.** A layer never goes upward ($t_{i+1,j} \geq t_{i,j}$).
- **Spatial consistency.** The inner layers are faster than the layers close to the cell walls ($t_{i,j+1} \geq t_{i,j}$).

Our approach is formulated as an optimization problem based on (1) in which we incorporate the layered image formation model and the above mentioned constraints. Besides, we align the whole input video at once in order to enforce the temporal consistency. We finally have:

$$\begin{aligned} &\min_{\mathsf{T}} \sum_{i=1}^{m} \sum_{(x,y)\in\Omega} \left(I_i(x,y) - \sum_{j=1}^{n} L_j\left(x, y - t_{i,j}\right) \right)^2 \\ &\text{subject to} \begin{cases} t_{i,j} \geq 0 & \forall i \in \{1,\ldots,m\}, \forall j \in \{1,\ldots,n\} \\ t_{i+1,j} \geq t_{i,j} & \forall i \in \{1,\ldots,m-1\}, \forall j \in \{1,\ldots,n\} \\ t_{i,j+1} \geq t_{i,j} & \forall i \in \{1,\ldots,m\}, \forall j \in \{1,\ldots,n-1\} \end{cases} \end{aligned} \tag{2}$$

where T is an $m \times n$ matrix containing the translation parameters. Problem (2) may be solved using for instance sequential quadratic programming. In practice, we used Matlab's fmincon function.

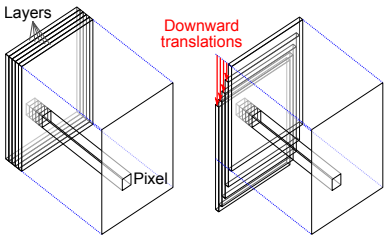

Fig. 3. The intensity of a pixel is the integration of the tracer concentration carried by the layers. A downward translation is associated to each one of the layers for each one of the input images. These translations are determined using our algorithm that extends DIR.

4 Results

The acquired video (visible in figure 6) shows a pattern shaped as a horizontal row. The acquisition frequency was 50 Hz with an exposure time of $\frac{1}{300}$ second. Classical background substraction and normalization techniques were applied to the raw data in order to correct the impurities of the HS cell and to compensate the variations of the illumination. Figure 4 shows the velocity profiles in the direction of the cell gap at four different times. The liquid has been discretized into 40 layers. The velocity profiles are computed from the whole row of marks. The profile computed for the image #8 corresponds to a time where the flow is in a transitory state. Figure 4 also shows that the profiles computed for images #44, #80, and #115 are well fitted by a parabola. Figure 5 shows a comparison of our approach with a ground truth measurement based on the position of the water/air interface. Figure 6 gives a 3D representation of the obtained results.

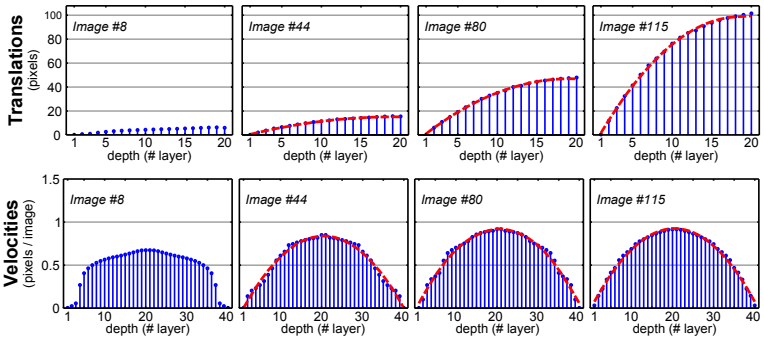

Fig. 4. Computed translations (top) at different times and corresponding velocities (bottom). Image #8 is taken when the PPF is developing whereas the other images correspond to an established PPF. A parabola is fitted when the steady flow is established (red curves): it shows that our results and the theoretical PPF are consistent.

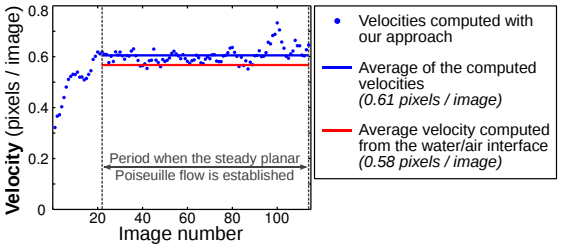

Fig. 5. Comparison of the velocities computed with our approach (averaged over the cell gap) and the average velocity computed from the water/air interface.

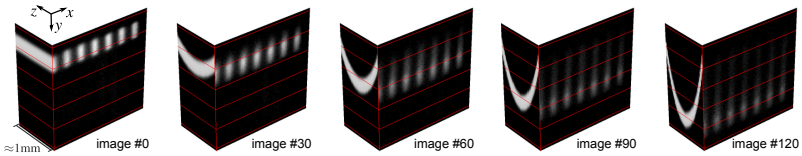

Fig. 6. 3D representation of the reconstructed volume for one of our video sequences. The acquired images are shown in the front of the volume (*xy* plane). The parabolic PPF computed with our approach is visible on the edge (*yz* plane) and its position is consistent with what is seen in front of the cell.

5 Discussion and Perspectives

We proposed an algorithm that can track the motion of a liquid in an HS cell being drained while simultaneously reconstructing the full volume of that cell from a single point of view. The photobleaching technique was extended to 3D measurements and is well suited for flows in confined environment with limited optical access. We extended DIR with a volumetric model and geometric constraints. These constraints were general assumptions on the physical model so as to make significant measurements of the PPF in a confined environment.

The technique we proposed may be useful to experimentally study other types of 3D motions in HS cells with a large field of view. Indeed, the principle of the superposed layers is quite generic and so is the volumetric image formation model we proposed. Our approach may be re-used by simply changing the deformation model (for instance by replacing the downward translation by local Euclidean transformations or even more complex deformation models). In particular, we are interested to measure the Lagrangian motion around a rising bubble in an HS cell.

References

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