Simultaneous Image Registration and **Monocular Volumetric Reconstruction of a Fluid Flow**

1. Context and Objective



- Computer vision applied to experiments in fluid mechanics
- Densely measure the Laminar Poiseuille Flow in an Hele-Shaw cell
- Pattern printed in the liquid using molecular tagging
- Tracking and volume reconstruction by combining:
- direct image registration
- a volumetric image formation model
- Use as few physical assumptions as possible



- Truly non-invasive
- A molecular tracer (fluorescein) is uniformly mixed to the water
- When excited with a `green laser', the tracer becomes fluorescent
- The fluorescence can be inhibited using a powerfull `blue laser'





[Image Interpolation]

- Bilinear and bicubic interpolation introduce a systematic bias
- Problem solved by fitting a **regularized B-spline** to the image (the images are thus considered as continuous functions)







Deforme pattern



3. Direct Image Registration

When the **brightness constancy assumption** is satisfied then direct image registration consists in minimizing the intensity difference between the input image (I_i) and the reference image (R):

$$\min_{t} \sum_{\mathbf{q} \in \Omega} \left(I_i(\mathbf{q}) - R(\mathbf{q} - (0, t)) \right)^2$$



In our case, the brightness constancy assumption is **not** satisfied due to the image formation model and the nature of the observed flow:







What we would observe with a uniform motion along the z-direction of the cell





Bicubic **B-spline**

4. Simultaneous Registration and **Volumetric Reconstruction**





Key idea: consider that the volume is made of layers moving independently from each other.

 $L_j(\mathbf{q}) =$

 $I_i(\mathbf{q}) \approx$

Additional hypotheses

Symmetry. The flow is symmetric with respect to the centre of the cell in the z-direction.

Positivity. All the translations are downward translations. **Temporal consistency.** A layer never goes upward. **Spatial consistency.** The inner layers are faster than the layers close to the walls of the cell.

Final optimization problem

 $\min_{\mathbf{T}} \sum_{i=1}^{m} \sum_{\mathbf{q} \in \Omega} \left(I_i(\mathbf{q}) - \sum_{j=1}^{n} L_j(\mathbf{q}) \right)$ subject to $\begin{cases} t_{i,j} \ge 0 & \forall i = 1, \dots, i \\ t_{i+1,j} \ge t_{i,j} & \forall i = 1, \dots, i \\ t_{i,j+1} \ge t_{i,j} & \forall i = 1, \dots, i \end{cases}$



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$$= \frac{1}{n} R(\mathbf{q})$$

$$= \sum_{j=1}^{n} L_j (\mathbf{q} - (0, t_{i,j}))$$

$$(\mathbf{q} - (0, t_{i,j})) \\ (m, \forall j = 1, \dots, n) \\ (m, \forall j = 1, \dots, n) \\ (m, \forall j = 1, \dots, n)$$



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