Simultaneous Image Registration and Monocular Volumetric Reconstruction of a Fluid Flow

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Overview. We propose to combine image registration and volumetric reconstruction from a monocular video of a draining off Hele-Shaw cell filled with water. A Hele-Shaw cell is a tank whose depth is small (*e.g.* 1 mm) compared to the other dimensions (*e.g.* 400 × 800 mm^2). We use a technique known as molecular tagging which consists in marking by *photobleaching* a pattern in the fluid and then tracking its deformations. The evolution of the pattern is filmed with a camera whose principal axis coincides with the depth of the cell. The velocity of the fluid along this direction is not constant. Consequently, tracking the pattern cannot be achieved with classical methods because what is observed is the integration of the marked molecules over the entire depth of the cell. The proposed approach is built on top of classical direct image registration in which we incorporate a volumetric image formation model. It allows us to accurately measure the motion and the velocity profiles for the entire volume (including the depth of the cell) which is something usually hard to achieve. The results we obtain are consistent with the theoretical hydrodynamic behaviour for this flow which is known as the *laminar Poiseuille flow*.

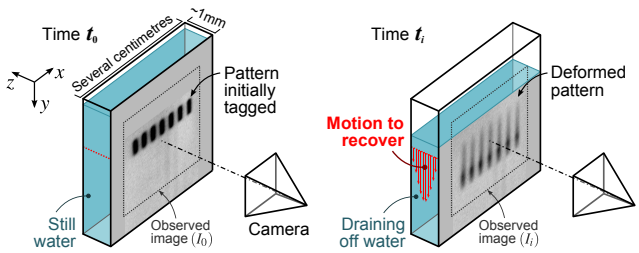

Figure 1: In this paper, we propose a new approach to register 2D images of a draining-off Hele-Shaw cell while simultaneously recovering the 3D motion of the fluid from a single video. The proposed approach relies on direct image registration. It uses as few physical assumptions as possible so as to experimentally validate the theoretical flow model.

The interest of this work is twofold. In computer vision, it is a new algorithm for monocular volumetric reconstruction. In fluid mechanics, it allows us to experimentally measure the Laminar Poiseuille Flow which is the expected theoretical model for this flow in a narrow confined environment.

Direct image registration is not suitable for tracking the motion. If we naively consider that the motion is a simple global downward translation, then one might think that standard direct image registration may be sufficient to track the motion of the pattern. However, as figure 2 illustrates, the brightness constancy assumption is not satisfied. This stems from the fact that the motion of the fluid accross the depth direction of the cell is not uniform. The consequence of that is that we observe 'streaks' in the trail of the marked pattern.

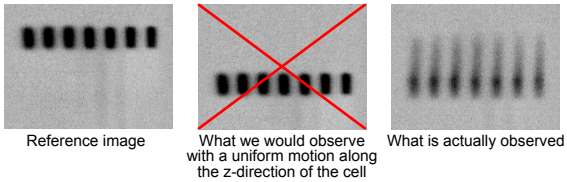

Figure 2: A simple tracking of the marked pattern is not possible stemming from the fact that the variation of the motion of the liquid across the depth of the cell is not negligible.

The proposed approach. The key idea of our approach is to combine direct image registration with a deformation model accounting for the underlying physical model of the flow. For that purpose, we consider that the volume is divided into n layers parallel to the cell walls (see figure 3). A translation parameter is associated to each one of these layers and for each one of the image of the video sequence. These translations are determined by minimizing a cost function inspired by the one classically used in direct image registration.

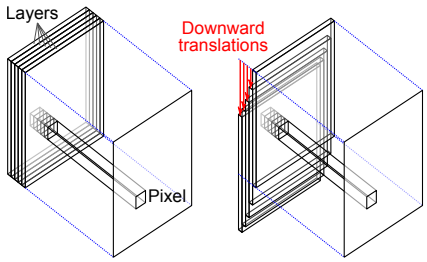

Figure 3: In our model, the intensity of a pixel is the integration of the tracer concentration carried by the layers. A downward translation is associated to each one of the layers for each one of the images of the video sequence. These translations are the unknown of our problem. They are determined using our algorithm built upon direct image registration.

In addition to our layered model, we included some supplementary constraints on the translations:

- **Symmetry.** The flow is symmetric with respect to the centre of the cell in the z-direction.
- **Positivity.** All the translations are downward translations.
- **Temporal consistency.** A layer never goes upward.
- **Spatial consistency.** The inner layers are faster than the layers close to the cell walls.

Our approach allows us to make a full volumetric reconstruction of the flow *i.e.* the motion of the fluid is recovered in both the cell plane and the depth direction. Figure 4 gives a 3D representation of the obtained results. Others experiments comes with the full version of the article.

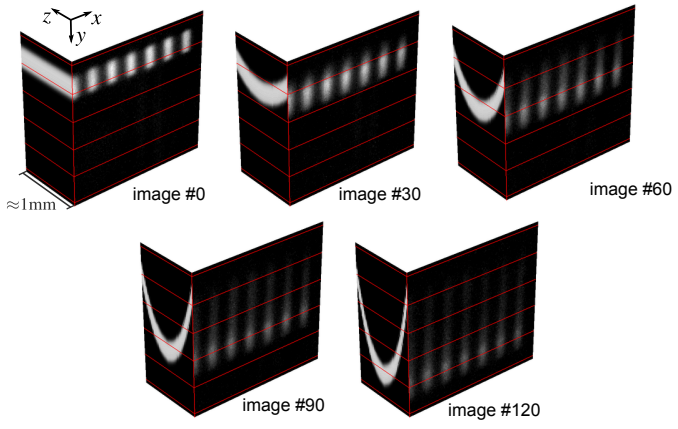

Figure 4: 3D representation of the reconstructed volume for one of our video sequences. The acquired images are shown in the front of the volume (xy plane). The parabolic laminar Poiseuille flow computed with our approach is visible on the edge and its position is consistent with what is seen in front of the cell.